**Air Force Institute of Technology**

**Graduate School of Engineering and Management**

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**CSCE 532 Automata and Formal Languages**

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# Day 3 – Finite Automata (Cont.) and nondeterminism

§1.1 Finite automata (cont.)

### Example (Sipser Exercise 1.36)

Let . Show that for each , the language is regular.

#### Solution

Proof: Let where be a DFA. Then accepts strings in , and rejects all others [how would you prove this?]. That is, , so is regular.

### Definition

Let and be strings and let be any language. We say that and are **distinguishable by**  if some string exists whereby exactly one of the strings and is a member of . Otherwise, for every string , we have exactly when and we say that and are **indistinguishable by** , and we write .

### Example

Let , , and as defined in the previous example. Then if , we have and , so and are distinguishable by .

Now let , , , and . If then and . Otherwise, so , , and thus exactly when , i.e. .

### Theorem

is an equivalence relation.

#### Proof

Let be a language. If then , so exactly when , i.e. . Next, if exactly when then exactly when , i.e. if then . Finally, if exactly when and exactly when , then exactly when , i.e. if and then . We have shown that is reflexive, symmetric, and transitive, i.e. an equivalence relation.

### Definition

Definition. Let be a language and let be a set of strings. Say that is **pairwise distinguishable by**  if every two distinct strings in are distinguishable by . Define the **index** of to be the maximum number of elements in any set that is pairwise distinguishable by . The index of may be finite or infinite.

### Example

Let , , and . Then because and are distinguishable by (as defined above), is pairwise distinguishable by . So is , while is not because and are not distinguishable by . Furthermore, I claim that no set containing more than three elements is pairwise distinguishable by , so that the index of is .

Now let , , , and . Then each of the ’s is pairwise distinguishable by (e.g. but so and are distinguishable by via the suffix ). Furthermore, I claim that the index of is infinite.

### Myhill-Nerode Theorem

Let be a language

1. If is recognized by a DFA with states, then has index at most .
2. If the index of is a finite number , then is recognized by a DFA with states.
3. is regular iff it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.

#### Proof

See Sipser (p.98).

Interpretation: for regular languages, the index of a language is the number of equivalence classes of . Each equivalence class corresponds to a state of a minimal DFA that recognizes .

Regular Grammars

### definition

A **grammar** is a 4-tuple where

1. is a finite nonempty set called the **variables**;
2. is a finite nonempty set, disjoint from , called the **terminals**;
3. is a finite nonempty set of **rules**, each of the form ; and
4. is the **start variable**.

### Example

Let where , , and . Then is a grammar. It is common to specify grammars by writing only the rules, and in so doing combining all rules with the same LHS, e.g. .

### Terminology

If , , , and are strings of variables and terminals, and is a rule of the grammar, we say that **yields** , written . Say that **derives** , written , if or if a sequence exists for and . The **language of the grammar** (or **language generated by the grammar**) is .

### Example

In the previous example, , so

* yields
* derives
* .
* I claim that (as defined above).

### Definition

A grammar is said to be **right-linear** if all rules are of the form or where and . A grammar is said to be **left-linear** if all rules are of the form or where and . A **regular grammar** is one that is either right-linear or left-linear.

### Theorem

A language is regular iff there exists a regular grammar such that .

### Proof

See Linz.

§1.2 Nondeterminism

### Example (Sipser Exercise 1.7a)

Give the state diagram of an NFA with three states recognizing the language .

#### Solution

### Practice (LINZ §2.2 Exercise 9)

Give the state diagram of an NFA with three states recognizing the language .

#### Solution